flow inclination in the rest region is very good, but it also indicates that the normal force interaction term $N \sin \varepsilon$ is very sensitive to changes in β. Therefore, the computed correction may still contain errors of fairly high magnitude. A more exact way of obtaining accurate Magnus data is to obtain the side forces and moments at zero spin.

Conclusions

Wind-tunnel Magnus data obtained to date on canted fin configurations may be invalid in that a severe normal force interaction may be present. Although it may be possible in some cases to correct for this interaction, it is in general best to remove the interaction by subtracting the zero spin data at each angle of attack. This requires a system in which the model can be kept at zero spin during an angle of attack sweep of the configuration.

References

[1] Ragan, F. J., "Magnus Measurements on a Free Spinning Stabilizer," AIAA Paper 70-559, Tullahoma, Tenn., 1970.

[2] Platou, A. S., "Wind Tunnel Magnus Testing of a Canted Fin or Self-Rotating Configuration," Ballistic Research Labs., Memo Rept. 2143, AD No. 736369, Dec. 1971, U.S. Army Aberdeen Research and Development Center, Aberdeen Proving Ground, Md.

Determining the Nature of Instability in Nonconservative Problems

RAYMOND H. PLAUT*
Brown University, Providence, R.I.

Introduction

IN aeroelastic and other nonconservative problems, instability may occur either by divergence or flutter.[1] It is of practical interest to know which type of instability will occur; for example, if instability is of the divergence type a lower bound for the critical load often may be obtained.[2] In this Note a certain class of continuous elastic systems under nonconservative loading is considered. First, the slopes of the loading-frequency curves are determined, then necessary conditions for flutter and divergence instability are derived, and finally the application of these results to the determination of the nature of instability is discussed and an example is presented.

Analysis

Consider a system whose deflection modes $y_n(\mathbf{x})$ are governed by the differential equation

$$-\mu(\mathbf{x})\Omega_n y_n(\mathbf{x}) + \mathscr{E} y_n(\mathbf{x}) + P\mathscr{T} y_n(\mathbf{x}) = 0 \tag{1}$$

Received February 7, 1972; revision received March 15, 1972. This research was supported in part by the U.S. Army Research Office—Durham under Grant DA-31-124-ARO-D-270 and in part by the U.S. Navy under Grant NONR N00014-67-A-0191-0009. The author wishes to thank H. Leipholz of the University of Waterloo for several stimulating discussions on the topic.

Index category: Structural Stability Analysis.

* Assistant Professor of Applied Mathematics and Engineering (Research).

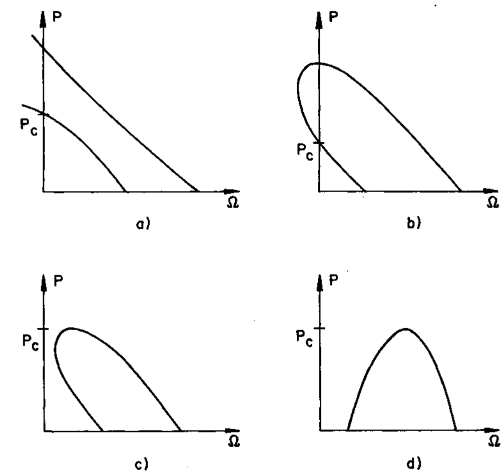

Fig. 1 Typical loading-frequency curves.

and the homogeneous boundary conditions

$$\mathscr{B} y_n(\mathbf{x}) = 0 \tag{2}$$

Here $P \geqq 0$ is the loading parameter, Ω_n is the frequency squared, $\mu(\mathbf{x}) > 0$ is the mass density, and $\mathscr{E}$, $\mathscr{T}$, and $\mathscr{B}$ are linear differential operators in terms of $\mathbf{x}$ and are independent of t and P. $\mathscr{E}$ is assumed to be self-adjoint and positive-definite. The stability of the equilibrium state $y_n(\mathbf{x}) \equiv 0$ is to be examined.

The condition for a nontrivial solution $y_n(\mathbf{x})$ of Eqs. (1) and (2) leads to a characteristic equation

$$F(\Omega_n, P) = 0. \tag{3}$$

Solution of Eq. (3) yields a loading-frequency relationship

$$P = p(\Omega_n) \tag{4}$$

Typical loading-frequency curves are shown in Fig. 1. At $P = 0$ the frequencies are real and positive and, assuming they are distinct, can be ordered by $0 < \Omega_1 < \Omega_2 < \ldots$. As P is increased the system may become unstable by divergence, as in Fig. 1 (a) and (b) at P_c, when one root Ω_n of Eq. (3) passes through zero to negative values, or by flutter, as in Fig. 1 (c) and (d) at P_c, when two roots merge and then become complex. Since Eq. (3) may be a complicated transcendental equation, it is useful to obtain some qualitative information on the properties of the loading-frequency curves.

The adjoint system to Eqs. (1) and (2) is defined by the equation

$$-\mu(\mathbf{x})\Omega_n z_n(\mathbf{x}) + \mathscr{E} z_n(\mathbf{x}) + P\mathscr{L} z_n(\mathbf{x}) = 0 \tag{5}$$

and boundary conditions

$$\mathscr{A} z_n(\mathbf{x}) = 0 \tag{6}$$

where the operators $\mathscr{L}$ and $\mathscr{A}$ are such that

$$\int z_n \mathscr{T} y_n \, d\mathbf{x} = \int y_n \mathscr{L} z_n \, d\mathbf{x} \tag{7}$$

whenever Eqs. (2) and (6) are satisfied. If Eq. (1) is multiplied by z_n and integrated over $\mathbf{x}$, one obtains

$$P = \left(\Omega_n \int \mu z_n y_n \, d\mathbf{x} - \int z_n \mathscr{E} y_n \, d\mathbf{x}\right) \Big/ \int z_n \mathscr{T} y_n \, d\mathbf{x} \tag{8}$$

This expression is stationary with respect to changes in y_n and z_n satisfying Eqs. (2) and (6), respectively. Differentiation thus yields the expression

$$dp/d\Omega_n = \int \mu z_n y_n \, d\mathbf{x} \Big/ \int z_n \mathscr{T} y_n \, d\mathbf{x} \tag{9}$$

for the slopes of the loading-frequency curves.

Substitution of Eq. (4) into Eq. (3) and differentiation of the resulting identity gives

$$\partial F/\partial \Omega_n + (\partial F/\partial P)\, dp/d\Omega_n = 0 \tag{10}$$

At a double root Ω_n the first term in Eq. (10) is zero, and assuming P is not concurrently a double root leads to

$$dp/d\Omega_n = 0 \tag{11}$$

From Eqs. (9) and (11) it follows that a necessary condition for flutter instability is given by

$$\int \mu z_n y_n dx = 0 \tag{12}$$

For the special case of a conservative system, $z_n = y_n$ and condition Eq. (12) cannot be satisfied except for $y_n(x) \equiv 0$. This verifies the fact that a conservative system cannot exhibit flutter instability.

An alternate slope expression is given by

$$dp/d\Omega_n = P\left[\Omega_n - \left(\int z_n \mathscr{E} y_n dx / \int \mu z_n y_n dx\right)\right]^{-1} \tag{13}$$

which can be obtained from Eqs. (1) and (9). At divergence $\Omega_n = 0$ and the slope is negative, so that

$$\int z_n \mathscr{E} y_n dx / \int \mu z_n y_n dx > 0 \tag{14}$$

is a necessary condition for divergence instability.

The slopes at $P = 0$ can often yield useful information. At $P = 0$ both y_n and z_n are equal to the modes of free vibration ξ_n, which are often tabulated, and Eq. (9) becomes

$$dp/d\Omega_n = \int \mu \xi_n^2 dx / \int \xi_n \mathscr{T} \xi_n dx \tag{15}$$

If these slopes are all negative, the system may become unstable by divergence as in Fig. 1(a) and (b) or by flutter as in Fig. 1(c). However a positive slope for $n = 1$ and a negative slope for $n = 2$ would indicate probable flutter instability as depicted in Fig. 1(d).

Example

Consider a uniform cantilevered column subjected to a partial follower load at its free end.[3] The governing equation is assumed to be

$$-\mu_0 \Omega y(x) + EIy''''(x) + Py''(x) = 0 \tag{16}$$

with boundary conditions

$$y(0) = 0,\ y'(0) = 0,\ y''(l) = 0,\ EIy'''(l) + (1-\eta)Py'(l) = 0 \tag{17}$$

where the subscript n has been deleted. For $\eta = 1$ the load acts tangentially and for $\eta = 0$ it acts in a vertical direction. The adjoint system has boundary conditions

$$z(0) = 0,\ z'(0) = 0,\ EIz''(l) + \eta P z(l) = 0,\ EIz'''(l) + Pz'(l) = 0 \tag{18}$$

and the same equation as Eq. (16).

After multiplying Eq. (16) by z and integrating with respect to x, the middle term must be integrated by parts to bring in the boundary terms which involve the load. One obtains

$$P = \left(\Omega \int_0^l \mu_0 zy\, dx - EI \int_0^l z''y''\, dx\right) \Big/ \left[\int_0^l zy''\, dx - (1-\eta)z(l)y'(l)\right] \tag{19}$$

which is stationary with respect to y and z, and this leads to the expression

$$\frac{dp}{d\Omega} = \int_0^l \mu_0 zy\, dx \Big/ \left[\int_0^l zy''\, dx - (1-\eta)z(l)y'(l)\right] \tag{20}$$

for the slopes of the loading-frequency curves. At $P = 0$ the slopes may be written as

$$\frac{dp}{d\Omega} = \int_0^l \mu_0 \xi^2 dx \Big/ \left[\eta \xi(l)\xi'(l) - \int_0^l (\xi')^2 dx\right] \tag{21}$$

where $\xi(x)$ denotes a free vibration mode of the column.

For $\eta = 0$ the loading is conservative and the slopes are negative for all P, giving divergence instability. For $\eta > 0.84$ the slope Eq. (21) is positive at the lowest frequency Ω_1, indicating flutter instability as in Fig. 1(d). As η increases from 0 to 0.84, a transition from divergence to flutter is therefore expected to occur. (The actual point of transition is $\eta = 0.5$, which was calculated in Ref. 3 by numerical solution of the transcendental characteristic equation.)

References

[1] Bolotin, V. V., *Nonconservative Problems of the Theory of Elastic Stability*, MacMillan, New York, 1963, pp. 1–23.

[2] Leipholz, H. and Huseyin, K., "On the Stability of One-Dimensional Continuous Systems with Polygenic Forces," SM Rept. 70, Jan. 1971, Solid Mechanics Div., Univ. of Waterloo, Waterloo, Ontario, Canada.

[3] Kordas, Z. and Zyczkowski, M., "On the Loss of Stability of a Rod Under a Super-Tangential Force," *Archiwum Mechaniki Stosowanej*, Vol. 1, No. 15, 1963, pp. 7–31.

Minimum Weight Passive Insulation Requirements for Hypersonic Cruise Vehicles

MARK D. ARDEMA*

NASA Ames Research Center, Moffett Field, Calif.

WEIGHTS of thermal protection systems for hypersonic vehicles are usually determined either from steady-state heat conduction analyses or, in more detailed studies, from numerical solution of the transient heat conduction equation. Analytical solutions of the heat equation with application to passive insultation systems for hypersonic cruise vehicles are not available in the standard heat conduction text[1] or elsewhere. This Note presents such analytic solutions for two representative cases.

It is assumed that the vehicle may be idealized as consisting of an exterior structure, a layer of passive insulation, and an interior structure which may contain fuel. Because fuel is consumed during the flight, a typical point on the interior structure will be in contact with the fuel for an initial portion of the flight but not in contact for the remainder. Consequently, there are two limiting cases of interest: 1) The wet wall case—the temperature at the interior wall is held to that of the boiling point of the fuel throughout the flight. (This corresponds to the bottom of the last tank to be emptied.) 2) The dry wall case—the heat transferred through the insulation is absorbed by the interior structure and the temperature of this structure is allowed to rise accordingly. (This corresponds to the top of the first tank to be emptied.)

The insulation thicknesses for each case are given by the solution to the two heat-transfer problems associated with these two cases. Vehicle geometry and tank sequencing must then be considered in estimating the fraction of vehicle surface area over which the individual thicknesses apply in the weight calculation. Since the variables of interest (the insulation thicknesses) cannot be obtained explicitly, it is necessary to solve for the thicknesses by iteration. The thicknesses are optimized in the sense that the weight of insulation plus fuel boiloff is minimized.

The major assumptions of the analyses are as follows: 1) heat transfer is by conduction only; 2) heat transfer tangential to the exterior surface is negligible compared with normal; 3) thickness

Received February 7, 1972.

Index categories: Aircraft Performance; Aircraft Structural Design (Including Loads).

* Aerospace Engineer, Advanced Concepts and Missions Division, OAST.